

Looking for skewness in financial time series ^(*)

Evidenze di asimmetria nelle serie storiche finanziarie

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Riassunto: In questo lavoro viene introdotta una procedura che permette di verificare la presenza di asimmetria in ambito finanziario. Il metodo viene applicato, assieme a procedure già presenti in letteratura, a 10 indici di borsa internazionali. Nello studio si considerano sia l'asimmetria marginale che quella condizionata. In particolare, si valuta anche la possibilità che l'asimmetria condizionata sia variabile nel tempo.

Keywords: skewness, conditional skewness, time-varying skewness, financial returns.

1. Introduction

The huge amount of work in financial time series has led to a general consensus of the scientific community about some empirical statistical features known as *stylised facts* (e.g., heavy-tailedness of financial distributions, and positive correlation among absolute returns), which have been thoroughly investigated.

On the other hand, relatively little work has been done to detect skewness of the return distribution. As a consequence, the occurrence of skewness, both unconditional and conditional, is still disputable and the empirical findings are not univocal. It should be also noted that in general unconditional symmetry does not imply conditional symmetry. The existence (or lack) of both unconditional and conditional symmetry is important in a number of situations. In particular, skewness is essential from a financial point of view since it may be considered as a measure of risk. For example, Kim and White (2004) stress that, if investors prefer right-skewed portfolios then, for equal variance, one should expect a “skew premium” to reward those willing to invest in left-skewed portfolios. With respect to optimal portfolio allocation, Chundachinda *et al.* (1997) showed that allocation can change considerably if higher than second moments are considered in selection. Along the same lines, Jondeau and Rockinger (2004) measured the advantages of using a strategy based on higher-order moments. Corrado and Su (1997) attributed the anomaly known as “volatility skew” in option pricing to the skewness and kurtosis of the returns distribution. In particular, concerning conditional skewness, Newey and Steigerwald (1997) showed that consistent estimation of the GARCH parameters can be obtained by QMLE if both the true and the assumed innovations densities are symmetric around zero and unimodal. When conditional symmetry does not hold, an additional parameter is necessary to identify the location of the innovation distribution. Furthermore, the assumption of conditional symmetry is commonly used in adaptive estimation and, in general, the shape of the conditional distribution may be crucial in any dynamical analysis such as, for example, dynamic optimal allocation or VaR estimation.

For all these reasons, we think it is worthwhile to study in more depth the occurrence of

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symmetry in time series of financial returns. We do this by following three steps, based on both tests and models. In the first step marginal symmetry is investigated by means of a suitable test; secondly, conditional skewness is analysed using a test and a non-Gaussian GARCH-type model. In both steps, skewness is assumed to be constant. The possibility of conditional time varying skewness is introduced in the third step, through a generalisation of the previous GARCH-type representation, that allows to dynamically model conditional variance, skewness and kurtosis.

2. Looking for skewness

The first step of our study consists in testing for unconditional skewness by means of the standardised third moment $S = \mu_3/\mu_2^{3/2}$, where μ_j is the j -th central moment. In this context, it should first be noted that the standard asymptotic test based on the relationship $\sqrt{n} \hat{S} \xrightarrow{d} N(0, 6)$ does not work correctly, either for dependent Gaussian or independent non-Gaussian data.

For this reason, the test proposed by Bai and Ng (2005) was adopted here. This test is based on the distribution of $\hat{S}$ and gives good results also for time series with leptokurtic marginal distribution. In particular it is found that, under the hypothesis of symmetry and without assuming independence or Gaussianity, $\sqrt{n} \hat{S} \xrightarrow{d} N(0, V)$, where $V = \alpha \Gamma \alpha' / \sigma^6$, $\alpha = [1, -3\sigma^2]$ and Γ is the 2×2 matrix defined as $\Gamma = \lim_{n \rightarrow \infty} n E(\bar{Z} \bar{Z}')$, with $\bar{Z}$ being the sample mean of

$$Z_t = \begin{bmatrix} (Y_t - \mu)^3 \\ (Y_t - \mu) \end{bmatrix}.$$

In this framework, the serial dependence in Y_t is explained through Γ , which represents the spectral density matrix of Z_t at frequency 0.

Bai and Ng (2001) also proposed a procedure for testing conditional symmetry. The test is distribution free and is based on the empirical distribution function of the estimated residuals $\{\hat{e}_t\}$ from a model specified as $y_t = f(I_{t-1}, \theta) + g(I_{t-1}, \theta) e_t$, where I_{t-1} is the information up to time $t - 1$. The statistic considered for the test is $CS = \max_y |S_n(y)|$, where $S_n(y) = \hat{W}_n(y) - \hat{W}_n(0) + u(y, e_t)$, with $\hat{W}_n(y) = \sum_{t=1}^n [I(\hat{e}_t \leq y) - I(-\hat{e}_t \leq y)] / \sqrt{n}$ and $u(y, e_t)$ is a suitable function defined in Bai and Ng (2001, p. 232). They proved that, under some technical assumptions and when conditional symmetry holds, $CS \xrightarrow{d} \max_{0 \leq s \leq 1} |B(s)|$, where $B(s)$ is a standard Brownian motion on $[0, 1]$. The test is shown to have asymptotic critical values at the 1%, 5% and 10% levels of significance equal to 2.78, 2.21 and 1.91, respectively.

The presence of skewness can also be assessed by using models able to represent it, which reduce to the symmetric case for given values of certain parameters. To evaluate the presence of constant conditional asymmetry we fitted the GARCH-type model

$$y_t = \mu_t + \delta_t, \quad t = 1, \dots, T, \quad (1)$$

where $\mu_t = E(y_t | I_{t-1})$ and the conditional distribution of $\delta_t | I_{t-1}$ is assumed to be a Pearson_{IV} (λ_t, a_t, v, m) random variable with density

$$f(\delta_t | \lambda_t, a_t, v, m; I_{t-1}) = C_t \left[1 + \left(\frac{\delta_t - \lambda_t}{a_t} \right)^2 \right]^{-m} \exp \left[-v \arctan \left(\frac{\delta_t - \lambda_t}{a_t} \right) \right]. \quad (2)$$

The terms λ_t , a_t , v and m are real-valued parameters that control the mean, variance, skewness and kurtosis of the distribution. In the present framework λ_t is chosen so that $E(\delta_t|I_{t-1}) = 0$. Thus, the model has constant conditional mean, skewness and kurtosis, while the conditional variance is time-varying. The dynamics of the conditional variance are described by modelling the evolution of the parameter a_t according to (3). The normalising constant C_t depends on m , v and a_t . A common choice in the literature, that we will follow, is to study the parameter m through $r = 2m - 2$.

In order to discuss a possible dynamic behaviour of conditional skewness an extension of this model was fitted. The framework of the generalised model is similar to the previous one, but now the conditional distribution of $\delta_t|I_{t-1}$ is $\text{Pearson}_{IV}(\lambda_t, a_t, v_t, m_t)$, with time-varying v and r . Inference on the dynamics of conditional moments is carried out by modelling the evolution of parameters a_t , v_t and $r_t = 2m_t - 2$, which is described by the following GARCH-type structure

$$a_t^2 = \omega_a + \alpha_a \bar{a}_{t-1}^2 + \beta_a a_{t-1}^2, \quad (3)$$

$$v_t = \omega_v + \alpha_v \bar{v}_{t-1} + \beta_v v_{t-1}, \quad (4)$$

$$r_t = \omega_r + \alpha_r \bar{r}_{t-1} + \beta_r r_{t-1}, \quad (5)$$

where $\bar{a}_t$, $\bar{v}_t$ and $\bar{r}_t$ are estimators of a_t , v_t and r_t based on the method of moments (see Heinrich, 2004, p. 10). These estimators are local in the sense that only the k more recent values of the series are used in their computation. For the results in the next section we chose $k = 10$.

In this formulation, model (1) - (5) has been introduced by Grigoletto and Lisi (2005) but some variants were proposed in literature (see e.g. Harvey and Siddique, 1999, and Brooks *et al.*, 2005). Modelling a_t , v_t and r_t rather than directly variance, skewness and kurtosis turns out to be easier because the latter quantities need to satisfy nonlinear constraints which are difficult to impose at each point in time, while the constraints concerning a_t , v_t and r_t can be imposed straightforwardly. The estimation of the ω_i , α_i and β_i , $i = a, v, r$, parameters is performed by maximum likelihood.

3. Empirical evidences

We look for empirical evidences of asymmetry by applying the previous methodologies to the daily returns, adjusted for split and dividends, of 10 major international stock indexes, namely the indexes CAC40, DAX, FTSE100, MIB30, Dow Jones, S&P500, Nasdaq, Hangseng, Nikkey225 and SMI. The time series refer to different periods, and have lengths between 1547 and 5483, but all end on December 13, 2005.

First, unconditional skewness was assessed with the test by Bai and Ng (2005). No evidence of asymmetry was found in the time series analysed. It should be remarked that an erroneous, in the sense described in Section 2, use of the distribution of $\hat{S}$ would have led to the conclusion that 9 of the 10 series show evidence of unconditional skewness.

In order to look for constant conditional skewness, we then applied both the Bai and Ng (2001) test and estimated the non-Gaussian GARCH type model defined in (2). In particular, the Bai and Ng (2001) test was applied to the standardised residuals of a suitable ARMA-GARCH model which includes a leverage effect used to represent the asymmetric effects that returns may have on volatility.

The results concerning conditional distributions differ from those on marginal skewness,

and show more evidence of asymmetry. In particular, conditional distributions were found to be significantly skewed with constant skewness in 5 cases using the Bai and Ng (2001) test and in 8 cases using the proposed model under the constraints $\alpha_v = \beta_v = \alpha_r = \beta_r = 0$. In the latter case, the significance of the asymmetry is a direct consequence of the parameters significance (at the usual 5% level).

We finally investigated the presence of dynamic conditional skewness by estimating the full model defined by (3)–(5). The significance analysis on the estimated parameters suggests that, at the 5% level, skewness is not constant in time in 6 cases. Only for the NIKKEY225 index all the analyses agree on the absence of any asymmetry. These results suggest that, in most cases, ignoring asymmetry may yield misleading conclusions.

In summary, the results on the 10 series analysed indicate that, even if unconditional skewness does not seem to be present, in several frameworks, like those described in the introduction, it is important to also investigate whether the conditional distribution is skewed. Besides, when skewness is time-varying, the use of models able to represent this behaviour may allow for a more realistic dynamic description of the financial quantities of interest.

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